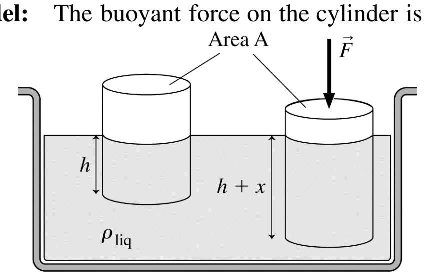
15.54. Model: The buoyant force on the cylinder is given by Archimedes' principle.

Visualize:



Solve: **(a)** Initially, as it floats, the cylinder is in static equilibrium, with the buoyant force balancing the cylinder's weight. The volume of displaced liquid is Ah, so

$$F_B = \rho_{liq}(Ah)g = w$$

Force F pushes the cylinder down distance x, so the submerged length is $h + x$ and the volume of displaced liquid is $A(h + x)$. The cylinder is again in equilibrium, but now the buoyant force balances both the weight and force F. Thus

$$F_B = \rho_{liq}(A(h + x))g = w + F$$

Since $\rho_{liq}(Ah)g = w$, we're left with

$$F = \rho_{liq}Agx$$

(b) The amount of work dW done by force F to push the cylinder from x to $x + dx$ is $dW = Fdx = (\rho_{liq}Agx)dx$. To push the cylinder from $x_i = 0$ m to $x_f = 10$ cm $= 0.10$ m requires work

$$W = \int_{x_i}^{x_f} F\,dx = \rho_{liq}Ag\int_{x_i}^{x_f} x\,dx = \tfrac{1}{2}\rho_{liq}Ag(x_i^2 - x_f^2)$$
$$= \tfrac{1}{2}(1000\ \text{kg/m}^3)\pi(0.020\ \text{m})^2(9.8\ \text{m/s}^2)(0.10\ \text{m})^2 = 0.616\ \text{J}$$